Final Exam Cosmology, 2012-2013

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The exam duration is 3 hours ($9^{00} - 12^{00}$).
The number of points given to each question is indicated next to it. The grade will be based on your answers to all questions.
Indicate clearly the steps in your solution and provide sufficient text.

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1. This question is composed of 5 short questions with 10 points each.

   (a) **What is the Deuterium bottleneck?**

   (b) **Explain why the neutrino and photon temperatures are different at $z = 0$.**

   (c) **We have shown in the class that the angular diameter distance for the benchmark model has a maximum at redshift of about 1.5. This implies that at larger distances a fixed size object will look larger. What in your opinion is the cause for this behavior?**

   (d) **Assuming the Universe contains only hydrogen, we have shown in the class that the decoupling redshift is $z_{dec} = 1200$. What would this redshift be if currently $T_{CMB} = 30$ K?**

   (e) **The difference between the mass of the neutron and that of the proton is about 1.29MeV. If this difference is 3 times smaller how this would affect the Big Bang Nucleosynthesis and which element then would be the most abundant in your opinion?**

(50 points)

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2. In the class we discussed the phase transition associated with recombination and decoupling.

   - **When Ω_b is larger does the recombination redshift become larger or smaller? Explain.**

- **Assume an instantaneous recombination at 1+z = 1300 in which the electron fraction X_e goes from 1 to 10^{-4} (NOT zero). Show that the universe is opaque before recombination and transparent after recombination .**
- **The universe gets later reionized due to the first galaxies and stars and X_e becomes unity again. What is the earliest redshift when this can happen without making the universe opaque again?**

(25 points)

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3. In the class we have written the Friedmann equation and solved it for several cases. Here we will consider the case of a universe with one component where this component has an equation of state parameter w.

   (a) **If the universe is flat what is the value of w that separates between a Universe with increasing rate of expansion to one with decreasing rate of expansion?**

   (b) **Show that for $w < -1$ the scale factor blows up to infinity at a certain time. Assume that the current time is t_0 and the current scale factor of the Universe, $a_0 = 1$.**

   (c) **Assume a de Sitter Universe, i.e, a flat universe that only has cosmological constant ($w = -1$). What is smallest scale factor of the Universe in such a model? Again assume that the scale factor today is 1.**

(25 points)

Good Luck!